CHAPTER

Exordium

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CHAPTER OUTLINE

With the development of science, technology and social productivity, human activities expand continuously into the space and ocean, and the research scope of the structural optimization extensively expands. Structural optimization design is becoming more and more important due to limited resources, intense engineering technological competitions, and environmental protection problems. Higher operating requirements are demanded for components of various high-precision and advanced devices. Designing structures and components to satisfy various constraints, therefore, provides both new opportunities and new challenges to structural engineers and mechanics researchers. On the other hand, the real-world simulations coupled in several physical fields are inevitably involved in structural and multidisciplinary optimization, greatly expanding the scope of structural optimization design.

Structural optimization aims at producing a safe and economic structural design subject to various load cases and structural materials. To obtain optimal design, not only mechanical properties such as strength, stiffness, stability, dynamic, and fatigue should be taken into account but also requirements of the application and operation such as manufacturing processes, construction conditions, and limits in the specifications of construction, manufacturing, and design should be satisfied. All requirements, conditions, and limits are expressed as constraints, whereas the economic index or a mechanical property is taken as the objective function. Design parameters, including design details such as the structure type and sizes, are taken as design variables. Henceforth, the optimization expression of a structural design is formed, and the mathematical model of the optimization can be further established. Finally, the optimization model is solved by optimization algorithms, and the optimal structure to satisfy the objective pursued by the user can be achieved automatically.

Structural optimization design is a synthetic subject involving computational mechanics, mathematical programming, computer science, and other engineering disciplines. It is highly comprehensive in theory and highly practical in method and technology; thus it is one of the important developments of the modern design method. Currently, applications of structural optimization design involve many fields, including aviation, aerospace, machinery, civil engineering, water conservancy, bridge, automobile, railway transportation, ships, warships, light industry, textile, energy, and military industry, to name just some. Engineering design problems should be solved properly, simultaneously pursuing better cost indicator of structure, the improvement of structure performances and enhancement on safety. Nonetheless, structural optimization design should meet the needs of the industrial production based on the accumulation of design experiences. Again, belonging to one of the synthesized and decision-making subjects, structural optimization design is founded on mathematical theory, method, and computer programming technology as well as its modeling technique.

In the 1960s, Schmit put forward the comprehensive design for structures by the mathematical programming. This marks the beginning of the structural optimization as an independent discipline. Hereafter, theory, method, and software of structural optimization design grew steadily. Over 50 years, the structural optimization has developed from the size optimization (or the so-called cross-section optimization in the initial stage), to the shape (or node) optimization, further to the topology optimization of skeletal structures, to the shape optimization and topology optimization of continuum structures. With a relative completion theoretical system formed and a great number of practical problems solved, huge economic and social benefits are created. However, the topology optimization design of continuum structures is still one of the hot spots due to emerging challenges from the lasting development and requirements of modern industry.

The authors believe that in the research of structural optimization design, engineering intuition and mechanical concepts should be closely combined with mathematical deduction; the analytic expression should be contrasted with geometrical intuition, which should be converted to an idea; and the conclusion of the low-dimensional space is sublimated to the high-dimensional space for rigorous developments of the theory in the structural optimization. Comprehensive, systematic researches on theory and numerical aspects should be carried out for the topology optimization of continuum structures. It is very important to grasp the key point, to hold the characteristic of the problem, and to analyze the essence through the phenomena during the researches.

In this chapter, the development history, the basic conception, and the classification of structural optimization are firstly summarized. Developments and methods of the topology optimization of continuum structures are then introduced. Finally, relevant mathematical theories involved with the research progresses in this monograph are presented.

1.1 RESEARCH HISTORY ON STRUCTURAL OPTIMIZATION DESIGN

1.1.1 CLASSIFICATION AND HIERARCHY FOR STRUCTURAL OPTIMIZATION DESIGN

Structural optimization optimizes the structural design. Since the 1960s, with the rapid development of computer technology and the finite element method, researches on how to provide a reliable and efficient method to improve the design of the structures for engineers have gradually become an important branch of mechanics. According to the feature of design variables, the structural optimization model can be classified into the model with continuous variables, the model with discrete variables, and the model with continuous and discrete mixed variables. According to the scope of the structural design variables, structural optimization design in general is divided into three levels (Fig. 1.1): size optimization, shape optimization, and topology optimization. These correspond to the detail design, basic design, and conceptual design phases of the product design, respectively.

Size optimization optimizes the sizes of components on the basis of specifying the structure type, topology, and shape. Its design variables can be the cross-sectional area of a rod, the thickness of a membrane or plate, a set of design parameters of a beam cross-section (such as the sizes of cross-section or quantities of a cross-section: area, bending moment of inertias in two directions, torsion moment of inertia, bending modulus, shear modulus, or torsion modulus), etc. [1]. Geometry optimization or shape optimization optimizes shapes of structural boundaries on the basis of specifying the structure type and topology. It belongs to the moving boundary problem. For continuum structures, structural boundaries are usually described by geometrical curves (such as line, arc, and spline) with a set of changeable parameters. The structural boundaries are adjusted when these parameters are changed. For truss structures, nodal coordinates are usually taken as design variables. The topology optimization changes structural topology in the design area to optimize a structural performance index and satisfy constraints on the stress, displacement, frequency, and so on under given loads and boundary conditions. For skeletal structures (including truss and frame), the

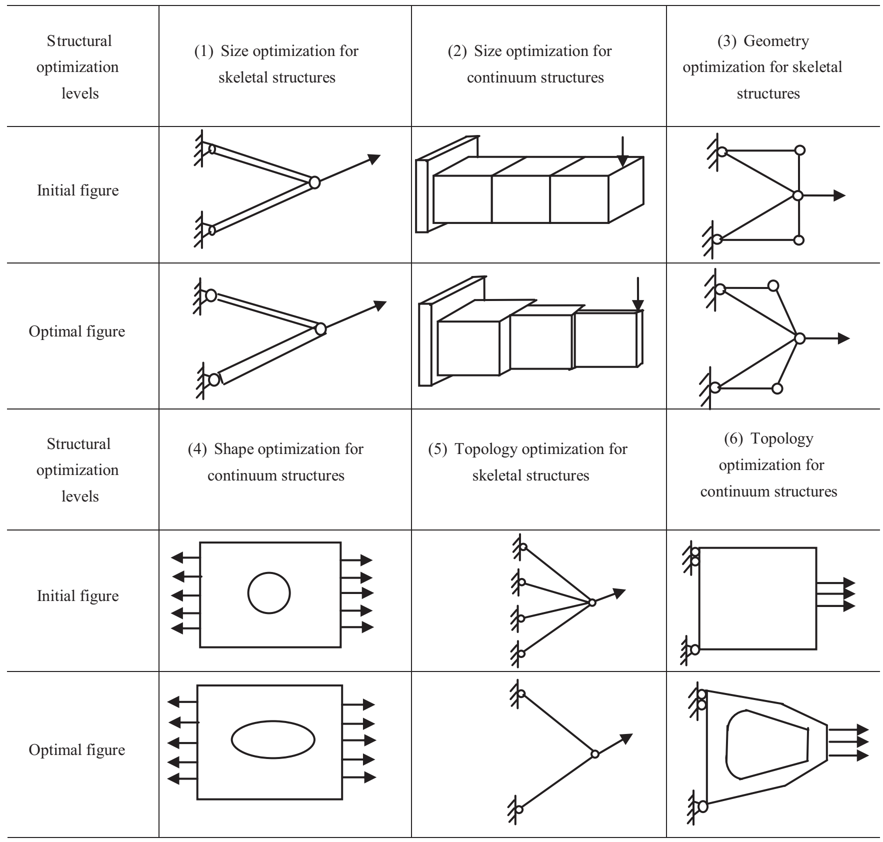

FIGURE 1.1

Levels of structural optimization.

presence or absence of nodes and components are taken as design variables. For continuum structures, the solid or void of subregions in the design is taken as a design variable.

Compared with the size optimization and geometrical optimization, the structural topology optimization not only has more undetermined parameters but also its topology variables have more influence on the optimization objective. Thus, greater economic benefits can be obtained. It is more attractive to engineering designers and has become a researching hot spot in the field of current structural optimization design. Due to design variables not being specific sizes or nodal coordinates, but the solid or void of subregions on the independent level, the difficulty of topology optimization is significant and is recognized as one of the most challenging topics in the field of current structural optimization [1−4]. Kirsch [5−11], who long engaged in the study of structural optimization design, considers the topology design problem to be the most difficult task in structural optimization. Optimization methods are still in the development stage. Applications of optimization methods in design practice are relatively fewer. This field urgently needs further improvement and development. The development of generic algorithms is still a challenge. Similar statements are also widely visible in the recent references [12−15].

The authors think it is very important to understand the structural topology optimization from the view of engineers. That is, the optimal topology of the topology optimization of continuum structures is in fact the reasonable paths of transferring loads and bearing responses. In the earlier researches, we understand it as the reasonable paths of transferring loads. The earlier understanding is intuitive for static topology optimization problems but is not precise enough for dynamic topology optimization problems. As a result, the understanding is revised in this monograph: the optimal topology of the topology optimization of continuum structures can be understood as the reasonable paths of transferring loads or the reasonable paths of bearing responses. By combining two aspects, it can be called succinctly the reasonable paths of transferring loads and bearing responses.

1.1.2 DEVELOPMENT OF STRUCTURAL OPTIMIZATION

The history of structural optimization can be traced back to Maxwell (1890)'s studies on the layout optimization of the truss. Thereafter, Michell [16] studied the layout optimization with stress constraints for the truss with coplanar forces applied to specified locations. The condition of the optimal truss with the lightest weight should be satisfied is obtained and later is called the Michell criterion. It is a milestone in the theory of structural optimization design. In essence, the Michell truss is a very advanced research in the field of structural topology optimization and still belongs to research directions at the highest levels.

The size optimization is the lowest level of optimization. Although it is the lowest level of structural optimization, it not only has the value of engineering application but also provides precious basic experiences for deeply understanding the structural optimization problem and various optimization algorithms. It was in 1960, 56 years after the Michell truss had been put forward, that structural optimization design became a subject. Schmit first established the mathematical model of the optimization design for elastic structures under multiple load cases [17] and put forward the solution method based on mathematical programming. Thereafter, a new phase of structural optimization design began. Why were there no followers at that time after Michell published his papers and the area became an advanced research? Why has Schmit published his papers, making a clarion call, caused numerous scholars to follow up immediately? The reason is the methodology and research tools. The idea of the criterion is the basis of Michell's method. He put forward the idea of material economic optimum for truss structures ("frame structures" is used in his paper; it should be "truss structures"). At that time, there are no the finite element method and the mathematical programming. But they are the basis of two methodologies—the mechanics analysis establishing the optimization model and the mathematical optimization solving the optimization model, which makes structural optimization design science.

If we say that Maxwell and Michell put forward an advantage direction at a high level when the foundation of structural optimization design had not yet been built, then Schmit's contribution is that he captured keenly the two methodologies constructing the architecture of structural optimization design just as they came out. There are many scholars following him. The architecture of structural optimization design is constructed layer by layer from size optimization to shape optimization and to topology optimization. It has the effect that someone raises his arm and is followed up. Except for two soft tools—two methodologies, there is an indispensable hard tool—the development of the computer. Thereafter, for the structural optimization problem with stress, displacement, and frequency constraints, various methods are adopted to solve it, including linear programming (LP), gradient projection, feasible direction, penalty function, and other methods. Because the mathematical programming theory is directly used to solve those problems and there is no clear conception of establishing the optimization model, the amount of calculation is huge during the iteration process. When directly using the mathematical programming theory, there is no high-efficiency algorithm due to lack of considering the mechanical characteristics of optimization problems. Thus, more effective ways to solve optimization problems are continually sought.

In 1968, Venkayya [18] and Gellatly [19] put forward the optimal criteria method. The iterative mode of design variables is selected according to the prescribed optimal criteria. The convergence is speeded up. Although this method is not rigorous in the theory aspect, its program is easy to implement, and the amount of calculation is small. Actually, the success of the criterion method is inevitable. When the mathematical programming method had not yet become an independent subject, mechanicians and engineers always put forward optimization criteria based on the mechanics conception or engineering intuition. Among various optimization criteria, there are the Michell truss criterion, the structure full stress criterion, and the component synchronous failure criterion as typical criteria. If we say that those early optimization criteria are perceptual criteria, then those optimization criteria appearing after 1969 can be called rational criteria. Accompanying the development of structural optimization criterion methods, structural optimization programming methods are developing continually.

In 1976, Schmit [20] divided design variables into several groups by the linking of design variables. The number of independent design variables was reduced. Invalid constraints were removed after every structure analysis, and the computational efficiency was improved. In 1979, Fleury first introduced the duality theory into the structural optimization problem [21]. By adopting the separable dual programming to solve the optimization problem, calculation results similar to the optimal criterion method were obtained. In 1980, Fleury and Schmit put forward the mixed optimality criteria method [22]. Some critical stress constraints were selected as effective stress constraints by using the virtual load method; other stress constraints were converted into upper and lower bounds. The number of iterations in the method has nothing to do with the number of design variables, thus the calculation efficiency is high. In every iterative process, the effective and noneffective constraints, the active and passive design variables are determined, the number design variables and the number of constraints are reduced, and therefore the computational efficiency is improved further. The development of structural optimization programming methods makes the number of iterations drop to the same level with structural optimization criterion methods. Why is there such a result? The key point is that the structural optimization approximation models were established, either consciously or unconsciously.

By turning back to look at the criterion method, we find that there is an approximated explicit optimization model lurking behind every criterion. Therefore, in the late 1970s, the structural optimization programming method and criterion method met. There are also some scholars who have yet to reach the confluence stage of both methods. For example, Khan put forward the strictest constraint method [23]. In every iteration process, based on structural stress analysis, the strictest constraint is picked out from all constraints. The design point is migrated to the strictest constraint plane by using the scaling step. Therefore, only an effective constraint needs to be considered in every iteration. The amount of calculation is reduced greatly. The strictest constraint method sometimes fails. For example, if the optimal point locates at two or more constraints at the same time, iterative oscillation will occur and the solution process will not be convergent.

Researchers in China have proposed many new methods. In 1973, in the symposium on the mechanics programming organized by the Chinese Academy of Sciences, Lingxi Qian presented the academic report "New developments on the optimization theory and method of the structural mechanics." This attracted extensive attention and responses in the mechanics and engineering field in China [2]. Since the 1980s, for the minimizing weight optimization problem of complicated structures simulated by different types of finite elements, Lingxi Qian et al. have taken the reciprocal of the cross-sectional size as the design variable. The objective function is expressed by the second-order Taylor expansion. Constraints are expanded by linear approximations. The iterative mode of the design variable, including the Lagrange multiplier, is derived by using Kuhn–Tucker conditions. The nonlinear programming method and the design criterion method are combined [2,24]. The stress constraints and displacement constraints are dealt with separately. The number of structural reanalysis is reduced further. Lingxi Qian led a team in the Dalian University of Technology to develop "structural optimization design with multiple elements, multiple load cases, and multiple constraints—DDDU system" [25,26]. Combining the mechanics conception and the mathematical programming method, some traditional difficulties are overcome. The sequential quadratic programming (SQP) algorithm for structural optimization is developed.

In 1983, Guangyuan Wang and Da Huo put forward the structural two-phase optimization method [27,28]. In this method, structural optimization design is divided into two stages. In the first stage, the criterion conditions are fully satisfied. In the second stage, the lightest design of the structure is solved. Two stages iterate alternatively. Renwei Xia and Ming Zhou studied the dual algorithm on the basis of the second-order approximation of functions [29], and put forward the generalized intermediate variables approximation method for the geometry optimization of truss structures [30]. Yunkang Sui improved the Newton method and the dual method by using two-point rational approximation [31,32]. The most effective approximated analytical method and its approximation method are found by using curve optimization theory to replace linear optimization theory [33,34]. The sequence rational programming method of the nonlinear programming is studied by the equivalent LP problem and equivalent quadratic programming (QP) problem, respectively [35]. A convenient and practical rational approximation method is put forward. By taking advantage of the information in the previous iteration, the waste of the repeated analysis and information is avoided; the efficiency of the optimization algorithm is improved. Huanchun Sun et al. discussed the discrete structural optimization problems [36,37]. Templeman and Yates constructed the multisegment element for the rod element of truss structures. Each segment of the multisegment element corresponds to an area in the discrete set. The discrete area design variable is thus converted skillfully to the corresponding continuous rod length design variable. The optimization model is solved by LP [38]. Considering that the number of design variables increases dramatically after the discrete variable is converted to the continuous variable in the method, Yunkang Sui and Kejian Peng [39] modified the method to construct the two-segment element near the continuous optimal solution. Under the condition of the total length of the rod element remaining constant, the number of continuous rod length design variables is the same as the number of the discrete cross-section; thereby, the method by Templeman and Yates is improved. In addition, the above method cannot be applied on the beam element as its internal forces are changed along the elemental length. Yunkang Sui and Yongming Lin constructed the infinite combination of the infinitesimal multiple segment element, thus improving the conversion method by Templeman and Yates to be applicable for the discrete variable optimization of frame structures [40,41]. Yunkang Sui also extended the method to the discrete size optimization of structures simulated by arbitrary types of finite elements [1].

Early works regarding shape optimization began from researches carried out by Zienkiewicz and Compell in 1973. They took nodal coordinates as design variables to describe the shape of a dam structure. The isoparametric element is adopted in the structural analysis. The sequential linear programming (SLP) method is adopted to solve the shape optimization problem of the dam structure [42]. In the same year, Deslva adopted the same mathematical method and structure analysis method to optimize the disk shape of the turbine [43]. However, for the method of taking nodal coordinates as design variables, the scale of the solved problem is limited and the solution precision is also affected because the number of design variables is usually large. Therefore, some scholars have put forward the method described structural shape by some fixed function. For example, in 1974, Vitiello took coefficients of the polynomial function as design variables, namely, a polynomial function was adopted to describe the distribution of thickness [44]. In this way, the scale of the problem was greatly reduced. Mature parameter optimization methods can also be adopted to solve the problem. Similar researches were also carried out by Ramarkishman [45]. The method of describing structural shape by a specified function adds an additional man-made constraint on the structures. It places the shape of the design structures in a fixed mode and can be selected at the optimum in small-scale changes. A more general method is to take functions as design variables. At first, the shape functions are defined. Then, the function to describe the initial shape of the structure is obtained from a linear combination of the shape functions with a set of undetermined parameters. Those undetermined parameters of the shape functions are taken as design variables. In 1979, Haug [46] proposed the variational principle of the shape optimization and solved the shape optimization problem of one-dimensional and two-dimensional plates.

In 1985, Haug and Choi proposed the material derivative sensitivity analysis method of shape optimization. The sensitivity analysis formula in the form of boundary integral and surface integral is presented [47]. In 1986, Belegundu developed the shape optimization method based on the natural design variable and the shape function [48]. A series of hypothetical loads applied to the structure are taken as natural design variables. The displacements produced by hypothetical loads are added to the initial shape to yield a new shape. The linear relationship between nodal displacements of meshes and the design variables produced in the finite element analysis is established. The shape optimization problem of elastic planes is solved. In 1987, Helder and Rodrganes studied the shape optimization of elastic body by the mixed variational formula. By adopting the mixed finite discretization method, a Euler–Lagrange formula based on the virtual work principle was put forward [49]. Oueau and Trompette studied the shape optimization problem of axisymmetric structures under symmetric or asymmetric loads. The objective function is to make local stresses along borders that are uniform and to reduce the stress concentration. The six- or eight-node isoparametric elements are used for the structural analysis. The algorithm cooperates with the automatic mesh generation program. Improvements on the derivative calculation of the stress and stiffness matrix were put forward. Parts of the shape of a helicopter rotor are optimized by the algorithm [50]. Ming Zhou and Rozvany combined the COC theory and the finite element method, and proposed an iterative COC algorithm. The problem with simple constraints can be solved very well, and the calculation efficiency is very high so that the large-scale problem can be solved [51,52].

In engineering practice, the optimization problem with multiple load cases often needs to be solved. In 1982, Botkin studied the structural shape optimization with multiple load cases [53]. The basic idea is to calculate and normalize the difference of the expected value and the calculated value of the strain energy density under various load cases separately in the same iteration process. The mesh updating area is determined according to those results. Meshes are automatically updated to prepare for the next iteration. For large-scale complex structures, only parts of the structural shape are often allowed to be changed. The whole structure is divided into many parts that are grouped into two categories: changeable parts and unchangeable parts. Only changeable parts need to be regenerated meshes in every iteration, which can reduce the amount of calculation of the finite element analysis and sensitivity analysis dramatically. In addition, the whole structure can be divided into several substructures (regions); internal degrees of freedom of the region are squeezed onto the boundary of the region, and then the relationship between the structure and the interface is established. In 1988, Huang and Huang put forward the substructure method of structural system optimization design [54]. In 1991, Botkin and Yang applied the substructure method of shape optimization of the three-dimensional solid [55].

There are also many scholars devoted to the research of the theory and method of the shape optimization in China. Yunkang Sui, Xicheng Wang, and Bei Wang put forward the secondary control method to overcome the difficulty of design variables controlling meshes in shape optimization software development. In the first stage, natural design variables determine coordinates of key points by the boundary shape function. In the second stage, key points determine coordinates of the mesh nodes by parameters coordinates of design variables of mesh nodes [25,26,56−58]. Yunkang Sui introduced some basic principles of mathematical programming into structural optimization and developed further from the view of methodology. According to the mapping inversion principle of the relationship, the dual algorithm of LP and geometric programming and the Lemke algorithm of QP were analyzed. The sequential mapping method is put forward to construct the algorithm solving the generalized QP [1]. Gengdong Cheng pointed out several difficulties existing in shape optimization. The shape optimization of the profile of the railway wheel and that of the turbine disc of an aero-engine were carried out [59]. Gengdong Cheng and Olhoff studied the shape optimization of the uniformity of the microstructure material [60]. Yuanxian Gu studied the sensitivity analysis of the shape optimization and the structural forming, and applied it to shape optimization with thermal stress constraints [61]. Jie Xing and Ping Cai developed the integrated software system FSOPD by using variational sensitivity analysis technology and the virtual load method. A shape optimization method was also put forward. The method takes the essential derivative and the variational sensitivity analysis as its foundation and defines the local and global velocity field. The integral of the sensitivity analysis is carried out in local regions, and the amount of calculation is reduced [62]. Weihong Zhang put forward the parameterized structural shape optimization design method by selecting automatically independent design variables [63,64], and a mesh disturbance physical analysis method was established based on finite element analysis. The sensitivity analysis method for size variables was established based on a simple proportion calculation [65]. The multiple