Chapter 1

Theoretical Foundation and Basic Properties of Thermal Radiation

Chapter Outline

All substances continuously emit and absorb electromagnetic energy when their molecules or atoms are excited by factors associated with internal energy (such as heating, illumination, chemical reaction, or particle collision). This process is called radiation. Radiation is considered a series of electromagnetic waves in classic physical theory, while modern physics considers it light quanta, that is, the transport of photons. Strictly speaking, radiation exhibits wave-particle duality, possessing properties of not only particles but also waves; this work

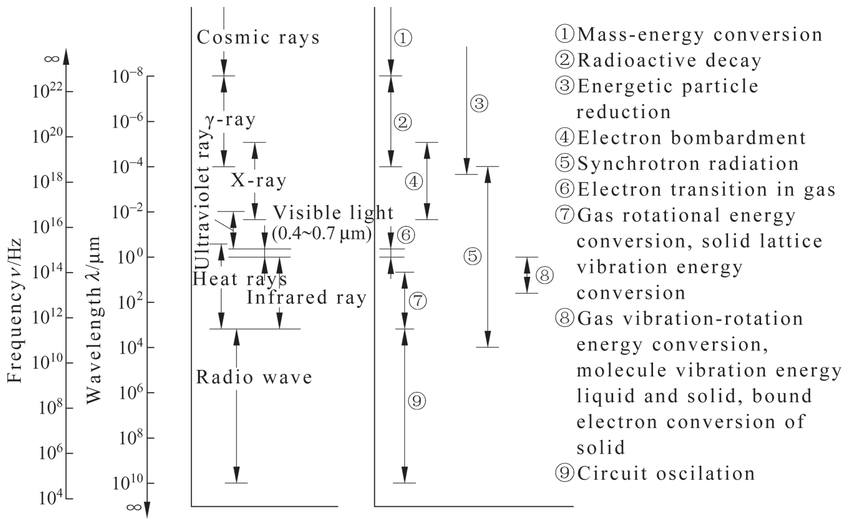

FIGURE 1.1 Electromagnetic wave spectrum.

considers these to be the same, that is, "radiation" refers simultaneously to both photons and electromagnetic waves.

At equilibrium, the internal energy of a substance is related to its temperature – the higher the temperature, the greater the internal energy. The emitted radiation covers the entire electromagnetic wave spectrum, as illustrated schematically in Fig. 1.1.

Thermal energy is the energy possessed by a substance due to the random and irregular motion of its atoms or molecules. Thermal radiation is the transformation of energy from thermal energy to radiant energy by emission of rays. The wavelength range encompassed by thermal radiation is approximately from 0.1 to 1000 μm, which can be divided into three subranges: the infrared from 0.7 to 1000 μm, the visible from 0.4 to 0.7 μm, and the near ultraviolet from 0.1 to 0.4 μm. Thermal radiation is a form of heat transfer between objects, characterized by the exchange of energy by emitting and absorbing thermal rays.

Consider, for example, two concentric spherical shells with different initial temperatures ($t_1 < t_2$) separated by a vacuum, as shown in Fig. 1.2. The temperature of sphere shell 2 increases as a result of heat exchange by thermal

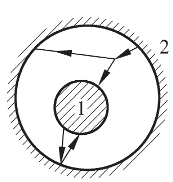

FIGURE 1.2 Radiation heat transfer between concentric sphere shells.

radiation between the two shells, since there is no heat conduction or heat convection between them.

This chapter will briefly outline the essential characteristics of thermal radiation, and the fundamental parameters that describe thermal radiation properties. The description of the basic laws of thermal radiation and the general methods used in thermal radiation transfer calculation are emphasized, as these are the theoretical foundation for solving heat radiation transfer problems and conducting related engineering calculations.

1.1 THERMAL RADIATION THEORY—PLANCK'S LAW [1,2,23,24]

At the end of the 19th century, classical physics had encountered two major roadblocks: the problem of relative motion between ether and measurable objects, and the spectrum law of blackbody radiation, that is, the failure of the energy equipartition law. The solution to the first problem led to relativity theory, and the second problem was solved after the establishment of quantum theory. Quantum theory also solved the problems of blackbody radiation, photoelectric effect, and Compton scattering.

In quantum mechanics, a particle's state at a definite time can be described by wave function $\Psi(r)$, and the motion of the particle can be described by the change of the wave function with time $\Psi(r,t)$. The wave function $\Psi(r,t)$ satisfies the following Schrodinger equation:

$$i\hbar \frac{\partial}{\partial t}\Psi(r,t) = \hat{H}\Psi(r,t) \tag{1.1}$$

where $\hat{H}$ is the Hamilton operator, and $\hbar$ is a constant.

In classical mechanics, if the Hamilton of a system is known, its Hamilton equation can be obtained to determine the motion of the entire system. For a quantum system, as long as the Hamilton operator $\hat{H}$ is known, the motion of the whole system can be determined, including its energy level distribution and transition. Only quantum mechanics can strictly and accurately describe the generation, transmission, and absorption characteristics of radiation. Strict description of thermal radiation, quantum mechanics, statistical physics, and other basic theories are necessary, however, these theories are too complex for engineering calculation, particularly concerning solutions to complicated motion system equations. The basic theory of macroscopic thermal radiation is also rather difficult to describe. To this effect, it is necessary to reasonably simplify and approximate problems for the convenience of engineering application. This task is performed by professional engineering disciplines.

This section focuses on blackbody radiation theory (Planck's law), the theoretical basis of thermal radiation. During the heat transfer process, "blackbody" refers to an object which can absorb all radiant energy of various wavelengths projected onto its surface. Planck's law describes the behavior of the blackbody, the derivation of which requires some basic concepts and methods of quantum mechanics and statistical physics. The following section provides a simple introduction to Planck's law and blackbodies, to help readers to understand the basic theory and history of thermal radiation.

According to quantum mechanics, the energy of a photon with frequency v is:

$$e = hv \tag{1.2}$$

where $h = 6.6262 \times 10^{-34}$ Js is the Planck constant.

According to statistical physics, the distribution with the greatest chance of a system consisting of a large number of particles is called "the most probable distribution." The most probable distribution is typically used to describe the equilibrium distribution of an isolated system. The photon does not obey the Pauli Exclusion Principle, thus it is a Boson; neutrons, protons, and electrons are called Fermions, as they all do obey the Pauli Exclusion Principle.

The classical particles satisfy the classical Maxwell–Boltzmann distribution under the conditions of continuous energy and degeneracy. Bosons obey Bose–Einstein distribution (B–E distribution), and Fermions obey Fermi–Dirac distribution.

According to the basic principle of statistical physics, the statistical equation of B–E distribution is derived as follows:

$$N_i = \frac{g_i}{\exp(\alpha + \beta e_i) - 1} \tag{1.3}$$

where N_i is the number of particles with energy level of $e_i = h\nu_i$, g_i is the degeneracy of the energy level e_i, and α and β are Lagrange factors. For a photon, $\alpha = 0$, and the statistical equation the photon obeys is:

$$N_i = \frac{g_i}{\exp(\beta e_i) - 1} \tag{1.4}$$

Consider a cavity with volume V and surface temperature T. For the photon in V:

$$\beta = \frac{1}{kT} \tag{1.5}$$

where k is the Boltzmann constant, equal to 1.38×10^{-23}J/K. The degeneracy of the photon in V at energy level e_i is:

$$g_i = \frac{8\pi V v_i^2}{c^3} dv_i \tag{1.6}$$

Substituting Eq. (1.5) and Eq. (1.6) into Eq. (1.4) provides the number of photons in the frequency range from ν_i to $\nu_i + d\nu_i$:

$$dN_i = \frac{8\pi V v_i^2}{c^3} \frac{dv_i}{e^{\frac{e_i}{kT}} - 1} \tag{1.7}$$

Substituting the photon energy $e_i = h\nu_i$ into Eq. (1.7) results in the following:

$$dN_i = \frac{8\pi V v_i^2}{c^3} \frac{dv_i}{e^{\frac{hv_i}{kT}} - 1} \tag{1.8}$$

Then, the energy of dN_i photons is:

$$de_i = hv_i dN_i = \frac{8\pi h V v_i^3}{c^3} \frac{dv_i}{e^{\frac{hv_i}{kT}} - 1} \tag{1.9}$$

The unit volume radiant energy density ranging from frequency ν_i to $\nu_i + d\nu_i$ is:

$$u_i dv_i = \frac{de_i}{V} = \frac{8\pi h v_i^3}{c^3} \frac{dv_i}{e^{\frac{hv_i}{kT}} - 1} \tag{1.10}$$

where u_i is monochrome radiation intensity, this is Planck's law in the form of radiant energy density. For a specified frequency ν_i, the subscript i can be removed. Eq. (1.10) can also be expressed by wavelength λ, because $\nu = c/\lambda$; thus, $d\nu = -\frac{c}{\lambda^2} d\lambda$, so radiant energy density ranging from wavelength λ to $\lambda + d\lambda$ is:

$$u_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \frac{d\lambda}{e^{\frac{hc}{\lambda kT}} - 1} \tag{1.11}$$

That is to say,

$$u_\lambda = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} \tag{1.12}$$

Obviously, Planck's law is an inevitable result derived from the basic theory of quantum mechanics and statistical physics which indicates that the radiant energy density ranging from $\lambda \sim \lambda + d\lambda$ only relates to wavelength λ and cavity temperature T, denoted as $u_\lambda = f(\lambda, t)$. From this law, the radiant energy density of different wavelengths at the same temperature T can be obtained, then the spectral distribution of radiant energy density can likewise be obtained. Planck's law is the basis of the entirety of thermal radiation theory. Thus, a good understanding of the law is necessary for mastering the properties and laws of thermal radiation.

For simplicity, "radiation" in the following chapters actually refers to "thermal radiation" unless otherwise specified.

1.2 EMISSIVE POWER AND RADIATION CHARACTERISTICS

A blackbody is an ideal radiation absorber and emitter. Similar to an "ideal gas" in thermodynamics, the blackbody is an ideal concept which forms the criteria used to compare actual radiators. Similarly, Planck's law as derived in Section 1.1 describes the ideal spectral distribution of radiant energy, which is only related to temperature and wavelength. What is the difference, then, between a real object or surface and a blackbody, pertinent to radiant energy distribution and radiation characteristics?

1.2.1 Description of Radiant Energy [3,4]

Let's first build an accurate description of "radiant energy." Radiation exists in the form of photons, having wave-particle dualism, thus it can be described using electromagnetic wave theory. An electromagnetic wave obeys the Maxwell equation as following:

$$\begin{cases} \nabla \times E = -\dfrac{\partial B}{\partial t} \\ \nabla \times H = -\dfrac{\partial D}{\partial t} + J \\ \nabla \cdot D = \rho \\ \nabla \cdot B = 0 \end{cases} \tag{1.13}$$

where E is the electric field intensity in position X at time t, denoted as $E(X, t)$. Here, we use simplified representation. H is magnetic field intensity; $D = \varepsilon E$ is an auxiliary variable, ε is the dielectric constant, B is the magnetic induction intensity, J is the current density, and ρ is the charge density.

The energy flow density of an electromagnetic wave can be expressed by Poynting vector S as follows:

$$S = E \times H \tag{1.14}$$

For linear media, $B = \mu H$, where μ is a constant. From Eq. (1.13), the following wave equation can be obtained:

$$\begin{cases} \nabla^2 \boldsymbol{E} - \dfrac{1}{c^2}\dfrac{\partial^2 \boldsymbol{E}}{\partial t^2} = 0 \\ \nabla^2 \boldsymbol{B} - \dfrac{1}{c^2}\dfrac{\partial^2 \boldsymbol{B}}{\partial t^2} = 0 \end{cases} \tag{1.15}$$

For a specific frequency (monochrome, in this case), the formal solution to Eq. (1.15) is:

$$\begin{cases} \boldsymbol{E}(\boldsymbol{X}, t) = \boldsymbol{E}(\boldsymbol{X}) e^{-i\omega t} \\ \boldsymbol{B}(\boldsymbol{X}, t) = \boldsymbol{B}(\boldsymbol{X}) e^{-i\omega t} \end{cases} \tag{1.16}$$

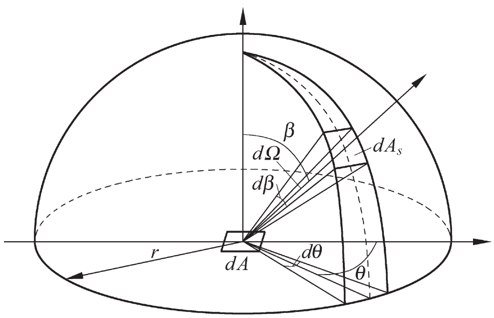

FIGURE 1.3 Definition of space of the solid angle.

where ω is the angular frequency of the electromagnetic field, and i is the imaginary number, $\sqrt{-1} = i$. For frequencies other than monochrome, the formal solution can be obtained similarly to Eq. (1.16) by Fourier analysis.

Combining Eqs. (1.14) and (1.15) results in the following:

$$\boldsymbol{S} = \boldsymbol{S}(\boldsymbol{X}, \omega) \tag{1.17}$$

Radiant energy is a function of position X and frequency ω. This is the start point of the calculation and analysis of radiant energy, and is a vital component of thermal radiation theory.

Physical variables are introduced below in order to describe the spiral distribution of radiant energy. As shown in Fig. 1.3, there is a differential area dA on the surface of a hemisphere with radius r, thus the space of the solid angle from differential area dA to sphere center O is defined as:

$$d\Omega = \frac{dA_s}{r^2} \tag{1.18}$$

The unit of solid angle Ω is the spherical degree (sr). Of course, the solid angle from the hemisphere to the sphere's center is 2π, and the solid angle from the entire sphere to its center is 4π. The direction angle is also required to describe the space properties – because this is basic information provided in any geometry course, it is not discussed in detail here.

Once direction angle and solid angle are defined, the concepts of radiation intensity and emissive power can be established.

1. Radiation intensity. As shown in Fig. 1.4, this is the radiant energy leaving a surface per unit area normal to the pencil of rays in unit time into the unit solid angle in a wavelength range from $0 \sim \infty$, denoted by I. The unit is: $\mathrm{W/(m^2 \cdot sr)}$.

   In Fig. 1.4, dA is a differential area in space, n is the normal line of dA, s is the radiation direction, dA_r is the projected area of dA normal to s, and β is

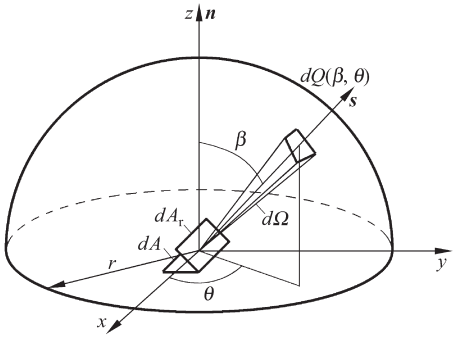

FIGURE 1.4 Definition of radiation intensity and radiation power.

the angle between s and n, namely, the direction angle in the radiation direction. Thus, $dA_r = \cos\beta dA$, where $d\Omega$ is the solid angle of any differential area in the direction s. According to these definitions, when the radiant energy in direction s is dQ, the following is obtained:

$$I\beta = \frac{dQ}{dA_r d\Omega} = \frac{dQ}{\cos\beta dA\, d\Omega} \tag{1.19}$$

where the subscript β in I_β denotes the direction.

2. Emissive power. As shown in Fig. 1.4, this is the energy leaving a surface per unit area in unit time into the unit solid angle described by β and θ, in a wavelength range from $0 \sim \infty$, denoted by $E_{\beta,\theta}$. The unit is $W/(m^2 \cdot sr)$. Typically, the surface radiation angle is irrelevant to θ, thus E_β represents emissive power, expressed as follows:

$$E_\beta = \frac{dQ}{dAd\Omega} \tag{1.20}$$

The relation between radiation intensity and emissive power is obtained as follows, by comparing Eq. (1.19) with Eq. (1.20):

$$E_\beta = I_\beta \cos\beta \tag{1.21}$$

Especially when considering the total radiation power from a surface to its hemispherical space, the hemispherical radiance E $W/(m^2)$ is appropriate. The relation between E and E_β is:

$$E = \int_0^{2\pi} E_\beta \, d\Omega \tag{1.22}$$

1.2.2 Physical Radiation Characteristics

The above section disclosed the characteristics that radiant energy is distributed according to space and frequencies; however, in engineering applications, we are more concerned about the radiation heat transfer between objects, such as between the hot flue gas and the water wall in a furnace, or the furnace wall and the work piece in a heating furnace. Therefore, it's still necessary to study the radiation characteristics of solid, liquid, and gas.

Objects have the ability to absorb, reflect, and penetrate external radiation, thus, absorptivity, reflectivity, and transmissivity are introduced to build a quantitative description of this ability. As shown in Fig. 1.5a, when an object receives external irradiation Q_I, some is absorbed, denoted as Q_α, some is reflected, denoted as Q_ρ, and some penetrates the object, denoted as Q_τ. See the following:

$$\text{Absorptivity}\ \alpha = Q_\alpha / Q_I \tag{1.23}$$

$$\text{Reflectivity}\ \rho = Q_\rho / Q_I \tag{1.24}$$

$$\text{Transmissivity}\ \tau = Q_\tau / Q_I \tag{1.25}$$

From the energy conservation relation shown in Fig. 1.5b, the following is obtained:

$$Q_I = Q_\alpha + Q_\rho + Q_\tau$$

Combining the above four equations results in:

$$\alpha + \rho + \tau = 1 \tag{1.26}$$

The above definitions suggest that absorptivity, reflectivity, and transmissivity are actually fractions of irradiation that has been absorbed, reflected, and penetrated. What are their properties, then?

Fig. 1.5 and Eqs. (1.23)–(1.25) demonstrate that the absorption, reflection, and transmission of an object relating to external projected energy are related to the characteristics of the object and the projected radiation. Generally, object properties, including physical properties, geometric structure (such as

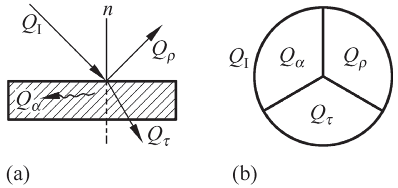

FIGURE 1.5 Absorption, reflection, and transmission of radiant energy. (a) Absorption, reflection, and penetration for radiation. (b) Energy conservation relationship.

species and the arrangement of atoms or molecules), temperature, and surface roughness all directly affect the absorption, reflection, and transmission of the irradiation, and therefore the frequency of irradiation. Basically, one object has different abilities to absorb, reflect, and penetrate radiation if the frequencies of projected energy are not the same.

Absorption: In quantum mechanics, the absorption of radiation is actually the process of energy level transition of particles that constitute the object (usually atoms or molecules) due to the absorption of photons under certain conditions. Because the micro states of particles with different species, temperatures, and atomic or molecular structures vary, the photon frequency/photon energy they absorb to satisfy energy level differences also vary, thus the absorption characteristics differ, and vice versa.

Reflection: Reflection is mainly dependent on the surface roughness of the object, and obeys reflection law.

Transmission: Transmission mainly occurs in gas media and is dependent on the absorption and reflection ability of the medium, which is further discussed in chapter: Emission and Absorption of Thermal Radiation.

According to the third law of thermodynamics, the temperatures of all objects are higher than 0 K, thus they all maintain thermal motion and emit thermal radiation. In order to describe the radiant energy emitted from different objects at different temperatures, the concept of emissivity is established with the blackbody as a criterion.

A blackbody is a hypothetical body that completely absorbs the radiant energy of all wavelengths from every direction. The absorptivity of the blackbody is 1 to all irradiation. The concept is the same as that of an ideal gas in thermodynamics—a blackbody is an ideal concept that does not exist in nature.

Object emissivity (also called "radiation rate") is defined as the ratio between the power emitted from the object and the power emitted from a blackbody with the same temperature as the object, denoted by ε:

$$\varepsilon = E/E_b \tag{1.27}$$

For a real object, $0 < \varepsilon < 1$. E_b is the emissive power of the blackbody, the calculation equation of which can be derived from Planck's law (see Section 1.3). It is important to note that though the blackbody appears black, an object with large emissivity is not always black. This happens because the human eye is only sensitive to visible light, while thermal radiation rays cover a broad wavelength including infrared. For example, the emissivity of black carbon at 52°C is 0.95~0.99 and this looks very black, but the emissivity of water at 0°C is 0.96~0.98 and water does not look black at all.

Other notable ideal concepts include transparent bodies, which have a transmission value of 1 for radiation from all wavelengths from any direction. Also mirror bodies, which have a reflection rate of 1 for radiation from all wavelengths from any direction. These comply with mirror reflection, as opposed